

Exercise 11

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$F(x) = \int_x^0 \sqrt{1 + \sec t} dt$$

$$\left[\text{Hint: } \int_x^0 \sqrt{1 + \sec t} dt = - \int_0^x \sqrt{1 + \sec t} dt \right]$$

Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

Rewrite $F(x)$ so that the variable is in the upper limit.

$$F(x) = - \int_0^x \sqrt{1 + \sec t} dt$$

As a result,

$$\begin{aligned} F'(x) &= - \frac{d}{dx} \int_0^x \sqrt{1 + \sec t} dt \\ &= -\sqrt{1 + \sec x}. \end{aligned}$$